**CHAPTER FIVE**

**FUNCTIONS AND ITS ASSOCIATED SIMPLIFICATION**

Simplification:

- Let x and y be two sets. When each number of the set x is associated or related to only one member of the set *y*, then such a relation is known as a function from x to y. - This is written as f: x → *y* and read as “the function from the set x to the set y or by the equation y = f(*x*). - The set x is known as the domain and the set y is known as the co-domain or the images. - The word function emphasizes the idea of the dependence of one quality on another. For example, let f be the mapping which is defined by f: *x*→ 2*x*+1, which can be written as y = 2*x* + 1. We say that y is a function of *x* which means that y depends on x - The variable x is called the independent variable, and y is called the dependent variable. The type of relation between *x* and y is called a functional relation. Each of the following defines the same set.

1. F: {*x*→2*x* – 1*, x* N}.



1. F = {(*x*,*y*): y = 2*x* – 1, *x* N}.



1. F = {*x*, 2*x* – 1: *x* N}.



1. Y = 2*x* – 1, *x*  N.



1. F(*x*) = 2*x* – 1, *x* N.

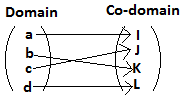


A function (or mapping) is therefore the relation between the elements of two sets, which are the domain and the co-domain, such that each element within the domain is associated or related to only one element in the co-domain.

Example (1)

Domain Co-domain

Example (2)



This is also a function, since each member of the mdomain is associated with only one member of the co-domain.

Example (3)

Domain Co-domain

This is not a function, for the first member of the domain i.e a, is associated with two members of the co-domain.

(Q1.) Given that F(*x*) = 2x+1, evaluated the following:

f(2) (b.) f(4) (c.) f(-3)

(d) f(-1) (e.) 2f(x) (f.) 5f(x).

Soln.

F(x) = 2x+1 =>

a.F(2) = 2(2)+1 = 4+1 = 5.

b. F(4) = 2(4)+1 = 8+1 = 9.

c. F(-3) = 2(-3)+1 = -6+1 = -5.

d. F(-1) = 2(-1)+1 = -2+1 = -1.

e. Since f(x) = (2x+1) ⇒ 2f(x) = 2(2x+1) = 4x+2.

f. 5f(x) = 5(2x+1) = 10x +5.

N/B: F(x) = 2x + 1 can be written as F(x) = (2x + 1) or F(x) = 1(2x+1).

(Q2.) If g(x) = 3x – 1, evaluate the following:

a.g(-1) b.) g(-2) c.) g(1/2)

d.) 3g(x) +1 e.) 4 g(x) – 2

f.) -2g(x)+2 g.) -3g(x) -3.

Soln.

g(x) = 3x – 1 =>

a.g(-1) = 3(-1) -1 = -3 – 1 = -4.

b. g(-2) = 3(-2) -1 = - 6 – 1 = -7.

c. g(1/2) = 3(1/2) -1 = 3 x 1/2 – 1 = 1.5 – 1 = 0.5.

d. g(x) = 3x – 1 => 3g(x) + 1 = 3(3x-1) + 1 = 9x – 3 +1 = 9x – 2 .

e. g(x) = 3x – 1 => 4 g(x) -2 = 4(3x - 1) -2 = (12x - 4) -2 = 12x - 4 - 2 = 12x – 6.

f. g(x) = 3x – 1 => -2 g(x) + 2 **=** -2(3x-1) + 2 = (- 6x+2) +2 = - 6x+2+2 = - 6x+4.

g. g(x) = 3x – 1 => -3g(x) -3 = -3(3x – 1) -3 = (-9x +3) -3 = -9x+3 -3 = -9x.

Q3. Given that f(x) = 2x + 1 and g(x) = 4x +2, evaluate the following:

a. g(x) +f(x) b. 2g(x) +f(x) c. 3g(x) + 4f(x)

d. ½ g(x) + 2f(x) e. g(x) – f(x) f. 3g(x) – 2(fx)

soln.

g(x) = 4x+2 and F(x) = 2x+1 =>

a.) g(x) + f(x) = (4x+2) + (2x+1) = 6x+3.

b.) 2g(x) + f(x) = 2(4x+2) + (2x+1) = 8x+4+2x+1 = 8x+2x+4+1 = 10x +5

c.) 3g(x) + 4f(x) = 3(4x+2) + 4(2x+1) = (12x+6) + (8x+4) = 12x +6+8x + 4 = 12x+8x+6+4 = 20x+10.

d.) ½ g(*x*) +2f(*x*) = 1/2(4*x*+2) +2 (2x+1) = ½ x 4x+1/2 x2 +4*x* +2 = 2*x*+1+4*x*+2 = 2x + 4x + 1+2 = 6x +3.

e.) g(*x*) – f(*x*)

= (4*x* +2) – (2*x* + 1) = 4*x*+2 – 2*x* – 1,

= 4*x* – 2*x* + 2 – 1 = 2*x* +1.

f.) 3g(x) – 2f(x)

= 3(4*x* + 2) -2(2*x* +1),

= 12*x* + 6 - 4*x* – 2 = 12*x* – 4*x* + 6 – 2

= 8*x +* 4.

Q4. Given that f(x) = -2x – 1 and g(x) = 3x – 2, evaluate the following: (i) f(x) + g(x)(ii) 2f(x) + 4g(x)

(iii) -2f(x) – g(x) (iv) -3f(x) + 2g(x)

(v) -2f(x) – 3g(x)

Soln.

F(x) = -2x – 1 and g(x) = 3x – 2 =>

(i) f(x)+ g(x) = (-2x – 1) + (3x – 2)

= -2x – 1 + 3x – 2 = -2x + 3x – 1 – 2

= x – 3.

(ii) 2f(x) + 4g(x) = 2(-2x – 1) + 4(3x – 2)

= -4x – 2 + 12x –8 = -4x + 12x – 2 – 8

= 8x – 10.

(iii) -2f(x) – g(x) = -2(-2x – 1) - (3x – 2) = 4x + 2 – 3x + 2 = 4x – 3x + 2 + 2

= x + 4

(iv) -3f(x) + 2g(x) = -3(-2x – 1) + 2(3x – 2) = 6x + 3 + 6x – 4.

= 6x + 6x + 3 – 4 = 12x – 1.

(v) -2f(x) – 3g(x) = -2(-2x – 1) – 3(3x – 2)

= 4x + 2 – 9x + 6 = 4x – 9x + 2 + 6

= -5x + 8.

Q5. Given that f(x) = 3x + 2 and g(x) = -4x – 2, evaluate the following.

a.) (i) f(-1) (ii) f(-2)

b.) (i) g(-1) (ii) g(-2) (iii) g(2)

c.) (i) f(x) + g(x) (ii) f(x) – g(x)

d.) (i) 2f(x) + 3 e.) 3f(x) – 2

f.) g(x) – f(x)

Soln.

a.) f(x) = 3x + 2 =>

(i) f(1) = 3(1) + 2 = 3 + 2 = 5.

(ii) f(-2) = 3(-2) + 2 = -6 + 2 = -4.

b.) g(x) = -4x – 2 =>

(i) g(-1) = -4(-1) – 2 = 4 – 2 = 2.

(ii) g(-2) = -4(-2) - 2= 8 – 2 = 6.

(iii) g(2) = - 4(2) – 2 = -8 – 2 = -10.

c.) (i) f(x) + g(x) = (3x + 2) + (- 4x – 2)

= 3x + 2 – 4x – 2 = 3x – 4x + 2 – 2

= -x + 0 = -x

(ii) f(x) – g(x) = (3x + 2) – (- 4x – 2)

= 3x + 2 + 4x + 2 = 3x + 4x + 2 + 2

= 7x + 4.

d.) 2f(x) + 3 = 2(3x + 2) + 3

= 6x + 4 + 3 = 6x + 7.

e.) 3f(x) – 2 = 3(3x + 2) – 2 = 9x + 6 – 2

= 9x + 4.

f.) g(x) – f(x) = (-4x – 2) – (3x + 2)

= - 4x – 2 – 3x – 2 = - 4x – 3x – 2 – 2

= -7x – 4.

Q6. If f(x) = 2x + 1, evaluate.

a. f(x + 1) b. f(2x + 3)

c. f(2x-1) d. f(3x-2)

Soln.

a. f(x) = 2x+1, => f(x+1)

= 2(x+1) +1 = (2x+2) + 1 = 2x + 2 + 1 = 2x + 3.

b. f(x) = 2x+1 => f(2x+3) = 2(2x+3) + 1 = (4x + 6) +1

= 4x + 6 + 1 = 4x + 7.

c. f(x) = 2x+1 => f(2x – 1) = 2(2x – 1) + 1 = (4x – 2) + 1

= 4x – 2+1 = 4x – 1.

d. f(x) = 2x+1 => f(3x – 2) = 2(3x – 2)+1 = (6x – 4) + 1

= 6x – 4 + 1 = 6x – 3 .

Q7. Given that g(x) = x – 2, evaluate the following: a. g(3x+1) b. g(-2x+1)

c. g(-4x – 3) d. g(2x – 1)

Soln.

a. g(x) = x – 2 => g(3x+1) = (3x+1) - 2 = 3x+1 – 2

= 3x – 1.

b. g(x) = x- 2 => g(-2x+1)

= (-2x+1) – 2 = -2x + 1 – 2 = -2x – 1.

c. g(x) = x – 2 => g(-4x – 3) = (-4x – 3) -2 = (-4x – 3) – 2

= -4x – 3 – 2 = -4x – 5.

d. g(x) = x – 2 => g(2x – 1) = (2x – 1) – 2 = 2x – 3.

Q8. A function f: x →3x+2, is defined on the set x

= {-3,-2,-1, 0, 1, 2, 3, 4, 5, 6}.

a. Find the images of the following:

i. -3 ii. -1 iii. 2 iv. 5

b. Find the value of x for which

i. F(x) = 8 ii. F(x) =11 iii. F(x) = -4

Soln.

a) i. F:x→3x+2 and for the image of -3, put x = -3 => f(x) = 3x+2 => f(x) = 3(-3) +2 = -9+2 = -7.

ii. For the image of -1, put x = -1. From f(x) →3x+2 => f(x) = 3 (-1)+2 = -3+2 = -1.

iii. For the image of 2, put x = 2. From f(x) = 3x+2 => f(x) =3(2) +2 => f(x) = 6+2 = 8.

iv. For the image of 5 put x =5.F(x) = 3x+2 = 3(5) +2 = 15+2 = 17.

b) i. F(x) =3x+2. If f(x) = 8 => 8 =3x+2 => 8 – 2 = 3x, => 6 = 3x => 3x = 6, => x = 6/2 => x = 3.

ii. F(x) = 3x+2 and if f(x) = 11 => 11 = 3x+2, =>11 – 2 = 3x

⇒ 9 = 3x, => 3x = 9 => x = 9/3, => x = 3.

iii. F(x) = 3x+2 and if f(x) = - 4 => - 4 = 3x + 2, => - 4 -2 = 3x => 3x = - 6, => x = -6/3 => x = -2.

Q9. A function f: x → 8x+1 is defined on the set x

= {-1, 0, 2, 3, 4, 5}

a. Find the images of -1 and 3.

b. Find the value of x for which f(x) = 7.

Soln.

F(x) = 8x+1.

1. For the image of -1, put x = -1 => f(x) = 8(-1) +1 = - 8+1 = -7.

For the image of 3, put x = 3 ⇒f(x) = 8(3) +1 = 24+1 = 25.

b. F(x) = 8x+1. If f(x) = 7 => 7 = 8x+1 => 7-1 = 8x, ⇒ 6 = 8x => 8x = 6, => x = 6/8 = 0.75.

Q10. A function f(x) = is defined on the set of real numbers.

a. Determine the images of the following:

i. -2 ii. -1 iii. 2 iv. 4

b. Evaluate the following:

i. f(3) ii.f(6)

c. Find the value of x for which f(x) is undefined.

Soln.

a. f(x) = 5x – 2

2x + 1

i. For the image of -2, put x = -2 => f(x) = 5(-2) – 2

= -10 - 2 2(-2) + 1

- 4+1

= -12 = 4

-3

ii. For the image of -1, put x = -1 =>f(x) =

=.

iii. For the image of 2, put x = 2 => f(x) = 5(2) – 2

= 2(2) + 1

= 8/5 = 1.6

b. i. f(x) = 5x – 2 => f(3) = 5(3) – 2

2x + 1 2(3) + 1

= 15 – 2 = 13 = 1.8.

6 + 1 7

ii. f(x) = 5x – 2, => f(6) = 5(6) – 2

2x + 1 2(6) + 1

= 30 – 2 = 28 = 2.1.

12 + 1 13

C For the value of x for which the function is undefined, put the down part to be equal to zero and solve for x.

i.e. 2x + 1 = 0 => 2x = 0 – 1,

=> 2x = -1 => x = -1 = -0.5.

2

The function is undefined when x = - 0.5.



Q11. A function f(x) = 3x+8 is defined on the set of real numbers. . x– 1

a. Find the image of -1 and 2.

b. Evaluate f(4).

c. Determine the value of x for which f(x) is undefined.

Soln.

a)f(x) = 3x + 8. For the image of -1, put x = -1

x – 1

=> f(x) = 3(-1) + 8

-1 – 1

= -3 + 8 = 5 = -2.5.

-2 -2

For the image of 2, put x = 2 => f(x) = 3(2) + 8 = 6 + 8

2 – 1 1

= 14.

b. f(x) = 3x + 8 => f(4) = 3(4) + 8

x – 1 4– 1

= 12 + 8 = 20

4 – 1 3

= 6.7.

c. Put x – 1 = 0 => x = 0 + 1, => x = 1 => the value of x for which f(x) is undefined is 1.

Q12. If g(x) = 2x – 1 and f(x) = 3x + 2, evaluate

a. g(1) + g(2)

b. g(1) + f(2)

c. f( -2) + g(3)

Soln.

a. g(x) = 2x – 1 → g(1) = 2(1) – 1 = 2 – 1 = 1.

g(2) = 2(2) – 1 = 4 – 1 = 3.

g(1) + g(2) = 1 + 3 = 4.



b. g(1) = 1. f(x) = 3x + 2 => f(2) = 3(2) + 2 = 6 + 2 = 8.

g(1) + f(2) = 1 + 8 = 9.



1. f(x) = 3x + 2 => f(-2) = 3(-2) + 2 = - 6 + 2 = - 4.

g(x) = 2x – 1 => g(3) = 2(3) – 1 = 6 – 1 = 5.

Therefore f(-2) + g(3) = - 4 + 5 = 1.

Q13. The function f is defined as f:(x) → 3x2 – 5x.

i. Evaluate f(-3).

ii. Find the value of x for which f(x) = -4/3.

Soln.

i. f : x → 3x2 – 5x => f(x) = 3x2 – 5x,

f (-3) = 3(-3)2 – 5(-3)



= 3(9) + 15 = 27 + 15 = 42.

ii. f(x) = 3x2 – 5x. If f(x) = - 4 then - 4 = 3x2 – 5x.

Multiply through using 3, 3 3

=> x 3 = 3x2 x 3 – 5*x* x 3,

=> - 4 = 9x2 – 15x,

=> 9x2 – 15x + 4 = 0, which is a quadratic in x, and by comparing this with ax2 + bx + c = 0

=> a = 9, b = -15 and c = 4.

To get the value of x, use the formula

Q14. Given f(x) = px + q, find the values of P and q, if f(2) = 4 and f(4) = 10.

Soln.

f(2) = 4, but f(x) = px + q => f(2) = p(2) + q

=>f(2) = 2p + q, but f(2) = 4

=> 4 = 2p + q

=> 2p + q = 4…………………………….eqn (1)

Also f(4) = 10, but f(x) = px + q

=> f(4) = p(4) + q

=> f(4) = 4p + q

But since f(4) = 10

=> 10 = 4p + q

=> 4p + q = 10…………………………..eqn (2)

Now solve eqn (1) and eqn (2) simultaneously.

2p + q = 4………….eqn (1)

4p + q = 10………...eqn (2)

Eqn. (2) x – 1 gives us - 4p – q = -10……………...eqn (3)

Add eqn (1) and eqn (3).

i.e. 2p + q = 4

+-4p – q = -10

-2p = - 6

-2p = - 6 => p = -6 = 3



-2

Put p = 3 into eqn (1) i.e 2p + q = 4

=> 2(3) + q = 4, => 6 + q = 4

q = 4 – 6 = -2.



Q15. The functions f and g are defined as f : x → 2 – 2x2 and g : x →

Evaluate i. g ii. f(2) (iii) g(3)

Soln.

i. g(x) → => g =

= = = - 0.8.

ii. f(x) = 2 – x2 => f(2) = 2 – 22

= 2 – 4 = -2.

g(x) = => g(3) =

= = 0.5.

=



Q16. Find the image of (-2, 4) under the mapping

→ .

Soln.

(x, y) = (-2, 4) → x = -2 and y = 4. Therefore →

→ => → =

=>.

=>The images of -2 and 4 are 8 and 10 respectively.

Q17. Find the images of (4, 1) under the mapping .

Soln.

(4, 1) => x = 4 and y = 1 and since ,

=>

The image of 4 = 5 and that of 1 = 3.

**Simplifying:**

To simplify, one must go through these step:

1. Find the L.C.M which is given by a x b = ab.

2. Divide ab using the a i.e.

3. Use the number above the a (ie 1) to multiply the b i.e 1 x b = b.

4. Then divide ab using the b i.e .

5. Use the number above the b (i.e 2) to multiply the a i.e 2 x a = 2a .

Hint

=>

To simplify

1.Find the L.C.M = 3a x b = 3ab.

2. Divide this using 3a i.e. .

3. Multiply the b with the 2 i.e b x 2 = 2b.

4. We next divide 3ab by b i.e.

5. Use the 3a to multiply the 4 i.e 3a x 4 = 12a.

Hint.

To simplify

1. Find the L.C.M which will be (a+b) x c = (a+b) c = c(a+b).

2. Divide by a + b i.e.

3.Multiply the c by the 2 i.e c x 2 = 2c.

4. Next divide c(a + b) by the c i.e.

5. Multiply a + b by the 5 i.e (a + b) x 5 = (a +b)5

= 5(a+b)

Hint. .

To simplify

1 Find the L.C.M which is given by 2a x (3x – 1) = 2a (3x – 1).

2. Next divide 2a (3x – 1) by 2a i.e = 3x -1.

3. Multiply 3x – 1 by x +1 i.e (3x – 1)(x + 1)

= 3x2 + 3x – x – 1 = 3x2 + 2x – 1.

4. Divide 2a (3x – 1) by 3x – 1 i.e

5. Multiply 2a by the 3 ie 2a x 3 = 6a.

Hint.

=



=

Q1. Simplify

Soln.

= .

Q2. Simplify

Soln

.

=

=

Q3. Simplify

Soln

=

= .

Q4. Simplify

Soln

= 2x + 2–(-)6

-2(x + 1)

= 2x+ 2 + 6

-2(x + 1) . . ..

QUESTIONS:

Q1. Giving that g(x) = -3x – 1, evaluate the following:

a. g(-2) Ans: 5

b. g(3) Ans: -10

c. g(-1) Ans: 2

d. g(5) Ans: -16

e. 2g(x) Ans: -6x – 2

f. 5g(x) Ans: -15x – 5

g. 2g(3) Ans: -20

h. 5g(2) Ans: -35

Q2. Given that f(x) = 4x + 2, evaluate

a. f(-1) Ans: -2

b. f(-2) Ans: -6

c. f(3) Ans: 14

d. 2f(x) Ans: 8x+4

e. 4f(x) Ans: 16x+8

Q3. If g(x) = 2x – 1 and f(x) = 3x+2, evaluate

A .g(x) + f(x) Ans: 5x +1

b. g(x) – f(x) Ans: -x – 3

c. f(x) – g(x) Ans: x +3

d. 2g(x) + 3f(x) Ans: 13x+4

e. 2g(x) – f(x) Ans: x – 4

f. 3f(x) – 2g(x) Ans: 5x + 8

Q4. Given that f(x) = 3x + 2, evaluate the following:

a. f(x + 1) Ans. 3x + 5

b. f(2x – 1) Ans. 6x – 1

c. f(x2 + 1) Ans.3x2 + 5.

d. f(2x) Ans. 6x + 2

e. f(3x + 2) Ans. 9x + 8

Q5. A function f : x → 2x – 1 is defined on the set

x = {-2, -1, 0, 2, 4, 5}.

Find the images of

a. -2 Ans. -5

b. -1 Ans. -3

c. 2 Ans. 3

d. 5 Ans. 9

Q6. Given that function f(x) = 2x – 3, evaluate

a. f(2) Ans. 1

b. f(6) Ans. 9

c. f(-2) Ans. -7

d. f(-1) Ans. -5

Q7. The function g(x) = is defined on the set of real numbers.

a. Determine the images of the following:

i. 3 Ans. 10

ii. 1 Ans. - 4

iii. -2 Ans. 1.25

iv. -3 Ans. 1.6

b. Evaluate the following:

i. g(4) Ans. 6.5

ii g(3) Ans. 10

iii. g(-5) Ans. 2

c. Determine the value of x for which the given function is undefined.

Ans. x = 2

Q8. Simplify +

Ans.

Q9. Simplify +

Ans.

Q10. Simplify Ans. .